# Cambridge Assessment
## Admissions Testing

Test of Mathematics for University Admission

Specification for November 2021

# Structure of the Test

The test will consist of two 75 minute papers, taken one after the other.

Each paper will consist of 20 multiple-choice questions.

Questions across the two papers carry equal weight and there will be no penalty for incorrect answers, so candidates are advised to attempt all questions.

There is no formulae booklet for this test; students are expected to understand and recall all relevant formulae.

Candidates may **not** use calculators.

The details of the papers are as follows:

**Paper 1: Mathematical Knowledge and Application**

Time: 75 minutes

Content: 20 multiple-choice questions

Requirements: Section 1 below

*This paper will test the candidate's ability to apply their mathematical knowledge in a variety of contexts. Candidates will be expected to know and use the mathematical content set out in Section 1 below.*

**Paper 2: Advanced Mathematical Thinking**

Time: 75 minutes

Content: 20 multiple-choice questions

Requirements: Sections 1 **and** 2 below

*This paper will test the candidate's ability to apply their conceptual knowledge to constructing and analysing mathematical arguments. For this paper candidates are expected to be familiar with the contents of Sections 1 and 2 below.*

# SECTION 1

*This section sets out the **mathematical knowledge** requirement for both papers of the test. The content of Part 1 is almost all covered within the pure mathematics specification of an AS level in mathematics, and the content of Part 2 is almost all covered within a Higher Level GCSE mathematics course.*

*There is some duplication of content across Parts 1 and 2.*

*Candidates are advised to read through these specifications carefully to ensure they are aware of all topics and areas that might be covered in the test.*

## Part 1

## Algebra and functions

**AF1**   Laws of indices for all rational exponents.

**AF2**   Use and manipulation of surds; simplifying expressions that contain surds, including rationalising the denominator; for example, simplifying $\frac{\sqrt{5}}{3+2\sqrt{5}}$, and $\frac{3}{\sqrt{7}-2\sqrt{3}}$.

**AF3**   Quadratic functions and their graphs; the discriminant of a quadratic function; completing the square; solution of quadratic equations.

**AF4**   Simultaneous equations: analytical solution by substitution, e.g. of one linear and one quadratic equation.

**AF5**   Solution of linear and quadratic inequalities.

**AF6**   Algebraic manipulation of polynomials, including:

- Expanding brackets and collecting like terms;
- Factorisation and simple algebraic division (by a linear polynomial, including those of the form $ax + b,$ and by quadratics, including those of the form $ax^2 + bx + c$);
- Use of the Factor Theorem and the Remainder Theorem.

**AF7**   Qualitative understanding that a function is a many-to-one (or sometimes just a one-to-one) mapping. Familiarity with the properties of common functions, including $f(x) = \sqrt{x}$ (which always means the 'positive square root') and $f(x) = |x|$

## Sequences and series

**SE1**   Sequences, including those given by a formula for the $n^{\text{th}}$ term and those generated by a simple recurrence relation of the form $x_{n+1} = f(x_n)$.

**SE2**   Arithmetic series, including the formula for the sum of the first $n$ natural numbers.

**SE3**   The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$.

**SE4**   Binomial expansion of $(1 + x)^n$ for positive integer $n,$ and for expressions of the form $\left(a + f(x)\right)^n$ for positive integer $n$ and simple $f(x)$; the notations $n!$ and $\binom{n}{r}$.

# Coordinate geometry in the $(x, y)$ plane

**CG1** Equation of a straight line, including $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; conditions for two straight lines to be parallel or perpendicular to each other; finding equations of straight lines given information in various forms.

**CG2** Coordinate geometry of the circle: using the equation of a circle in the forms $(x - a)^2 + (y - b)^2 = r^2$, and $x^2 + y^2 + cx + dy + e = 0$.

**CG3** Use of the following circle properties:

- The perpendicular from the centre to a chord bisects the chord;
- The tangent at any point on a circle is perpendicular to the radius at that point;
- The angle subtended by an arc at the centre of a circle is twice the angle subtended by the arc at any point on the circumference;
- The angle in a semicircle is a right angle;
- Angles in the same segment are equal;
- The opposite angles in a cyclic quadrilateral add to $180°$;
- The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

# Trigonometry

**TR1** The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.
The sine rule includes an understanding of the 'ambiguous' case (angle-side-side). Problems might be set in 2- or 3-dimensions.

**TR2** Radian measure, including use for arc length and area of sector and segment.

**TR3** The values of sine, cosine, and tangent for the angles $0°, \ 30°, \ 45°, \ 60°, \ 90°$.

**TR4** The sine, cosine, and tangent functions; their graphs, symmetries, and periodicity.

**TR5** Knowledge and use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.

**TR6** Solution of simple trigonometric equations in a given interval (this may involve the use of the identities in **TR5**); for example: $\tan x = -\frac{1}{\sqrt{3}}$ for $-\pi < x < \pi$;
$\sin^2\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$ for $-2\pi < x < 2\pi$; $12\cos^2 x + 6\sin x - 10 = 2$ for $0° < x < 360°$.

# Exponentials and Logarithms

**EL1**   $y = a^x$ and its graph, for simple positive values of $a$.

**EL2**   Laws of logarithms:

$$a^b = c \Leftrightarrow b = \log_a c$$

$$\log_a x + \log_a y = \log_a(xy)$$

$$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$$

$$k \log_a x = \log_a\left(x^k\right)$$

including the special cases:

$$\log_a \frac{1}{x} = -\log_a x$$

$$\log_a a = 1$$

Questions requiring knowledge of the change of base formula will not be set.

**EL3**   The solution of equations of the form $a^x = b$, and equations which can be reduced to this form, including those that need prior algebraic manipulation; for example, $3^{2x} = 4$ and $25^x - 3 \times 5^x + 2 = 0$.

# Differentiation

**DF1**   The derivative of $f(x)$ as the gradient of the tangent to the graph $y = f(x)$ at a point. In addition:

- Interpretation of a derivative as a rate of change;

- Second-order derivatives;

- Knowledge of notation: $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $f'(x)$, and $f''(x)$.

Differentiation from first principles is excluded.

**DF2**   Differentiation of $x^n$ for rational $n$, and related sums and differences. This might require some simplification before differentiating; for example, the ability to differentiate an expression such as $\dfrac{(3x+2)^2}{x^{\frac{1}{2}}}$ could be required.

**DF3**   Applications of differentiation to gradients, tangents, normals, stationary points (maxima and minima only), increasing $[\, f'(x) \geq 0 \,]$ and decreasing $[\, f'(x) \leq 0 \,]$ functions. Points of inflexion will not be examined, although students are expected to have a qualitative understanding of points of inflexion in the curves of simple polynomial functions.

# Integration

**IN1** Definite integration as related to the 'area between a curve and an axis.' Candidates are expected to understand the difference between finding a definite integral and finding the area between a curve and an axis. Integrals could be given with respect to $x$ or with respect to $y$.

**IN2** Finding definite and indefinite integrals of $x^n$ for $n$ rational, $n \neq -1$, and related sums and differences, including expressions which require simplification prior to integrating; for example, $\int (x+2)^2 \, dx$, and $\int \frac{(3x-5)^2}{x^{\frac{1}{2}}} \, dx$.

**IN3** An understanding of the Fundamental Theorem of Calculus and its significance to integration. Simple examples of its use may be required in the two forms,
$\int_a^b f(x) \, dx = F(b) - F(a)$, where $F'(x) = f(x)$, and $\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$.

**IN4** Combining integrals with either equal or contiguous ranges;
for example, $\int_2^5 f(x) \, dx + \int_2^5 g(x) \, dx = \int_2^5 [f(x) + g(x)] \, dx$,
and $\int_2^4 f(x) \, dx + \int_4^3 f(x) \, dx = \int_2^3 f(x) \, dx$.

**IN5** Approximation of the area under a curve using the trapezium rule; determination of whether this constitutes an overestimate or an underestimate.

**IN6** Solving differential equations of the form $\frac{dy}{dx} = f(x)$.

# Graphs of Functions

**GF1** Recognise and be able to sketch the graphs of common functions that appear in this specification: these include lines, quadratics, cubics, trigonometric functions, logarithmic functions, exponential functions, square roots, and the modulus function.

**GF2** Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$, with the value of $a$ positive or negative. Compositions of these transformations.

**GF3** Understand how altering the values of $m$ and $c$ affects the graph of $y = mx + c$.

**GF4** Understand how altering the values of $a$, $b$ and $c$ in $y = a(x + b)^2 + c$ affects the corresponding graph.

**GF5** Use differentiation to help determine the shape of the graph of a given function; this might include finding stationary points (excluding inflexions) as well as finding when the function is increasing or decreasing.

**GF6** Use algebraic techniques to determine where the graph of a function intersects the coordinate axes; appreciate the possible numbers of real roots a general polynomial can possess.

**GF7** Geometric interpretation of algebraic solutions of equations; relationship between the intersections of two graphs and the solutions of the corresponding simultaneous equations.

# Part 2

# Number

- Order, add, subtract, multiply and divide whole numbers, integers, fractions, decimals, and numbers in index form.
- Use the concepts and vocabulary of factor, multiple, common factor, highest common factor (hcf), least common multiple (lcm), composite (i.e. not prime), prime number, and prime factor decomposition.
- Use the terms square, positive and negative square root, cube and cube root.
- Use index laws to simplify, multiply, and divide integer, fractional, and negative powers.
- Interpret, order, and calculate with numbers written in standard index form.
- Understand equivalent fractions.
- Convert between fractions, decimals, and percentages.
- Understand and use percentage including repeated proportional change and calculating the original amount after a percentage change.
- Understand and use direct and indirect proportion.
- Use ratio notation including dividing a quantity in a given ratio, and solve related problems (using the unitary method).
- Understand and use number operations, including inverse operations and the hierarchy of operations.
- Use surds and $\pi$ in exact calculations; simplify expressions that contain surds, including rationalising the denominator.
- Calculate upper and lower bounds to contextual problems.
- Approximate to a specified and appropriate degree of accuracy, including rounding to a given number of decimal places or significant figures.
- Know and use approximation methods to produce estimations of calculations.

# Algebra

- Distinguish between the different roles played by letter symbols.
- Manipulate algebraic expressions by collecting like terms; by multiplying a single term over a bracket; by expanding the product of two linear expressions.
- Use index laws in algebra for multiplication and division of integer, fractional, and negative powers.
- Know and use of $\left(a^b\right)^c = a^{bc}$
- Set up and solve linear equations, including simultaneous equations in two unknowns.
- Factorise quadratics, including the difference of two squares.
- Simplify rational expressions by cancelling or factorising.
- Set up quadratic equations and solve them by factorising.
- Set up and use equations to solve problems involving direct and indirect proportion.
- Derive a formula, substitute into a formula.
- Change the subject of a formula.
- Solve linear inequalities in one or two variables.
- Generate terms of a sequence using term-to-term and position-to-term definitions.
- Use linear expressions to describe the $n^{\text{th}}$ term of a sequence.
- Use Cartesian coordinates in all 4 quadrants.
- Recognise the equations of straight lines; understand $y = mx + c$ and the gradients of parallel and perpendicular lines.
- Understand that the intersection of graphs can be interpreted as giving the solutions to simultaneous equations.
- Solve simultaneous equations, where one is linear and one is quadratic.
- Recognise and interpret graphs of quadratic functions, simple cubic functions, the reciprocal function, trigonometric functions and the exponential function $y = k^x$ for simple positive values of $k$.
- Construct linear functions from real-life problems; interpret graphs modelling real situations.

# Geometry

- Recall and use properties of angles at a point, on a straight line, perpendicular lines and opposite angles at a vertex.
- Understand and use the angle properties of parallel lines, intersecting lines, triangles and quadrilaterals.
- Calculate and use the sums of the interior and exterior angles of polygons.
- Recall the properties and definitions of special types of quadrilateral.
- Recognise and use reflectional and rotational symmetry of 2-dimensional shapes.
- Understand congruence and similarity.
- Use Pythagoras' theorem in 2-dimensions and 3-dimensions.
- Understand and construct geometrical proofs, including using circle theorems:
    - The perpendicular from the centre to a chord bisects the chord;
    - The tangent at any point on a circle is perpendicular to the radius at that point;
    - The angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
    - The angle in a semicircle is a right-angle;
    - Angles in the same segment are equal;
    - The opposite angles in a cyclic quadrilateral add to 180°;
    - The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.
- Use 2-dimensional representations of 3-dimensional shapes.
- Describe and transform 2-dimensional shapes using single or combined rotations, reflections, translations, or enlargements, including the use of vector notation.
- Understand and be able to use the standard trigonometric ratios: sin, cos, and tan.

# Measures

- Calculate perimeters and areas of shapes made from triangles, rectangles and other shapes.
- Find circumferences and areas of circles, including arcs, segments and sectors.
- Calculate the volumes and surface areas of right prisms, pyramids, spheres, cylinders, cones and solids made from cubes and cuboids (formulae will be given for the sphere and cone).
- Use vectors, including the sum of two vectors, algebraically and graphically.
- Use and interpret maps and scale drawings.
- Understand and use the effect of enlargement for perimeter, area, and volume of shapes and solids.
- Convert measurements from one unit to another, including between imperial and metric (conversion factors will be given for imperial/metric conversions).
- Knowledge of the SI prefixes milli (m), centi (c), deci (d), and kilo (k) when used in connection with any SI unit.
- Recognise the inaccuracy of measurement.
- Understand and use three-figure bearings.
- Understand and use compound measures.

# Statistics

- Identify possible sources of bias.
- Identify flaws in data collection sheets and questionnaires in an experiment or a survey.
- Group, and understand, discrete and continuous data.
- Extract data from lists and tables.
- Design and use two-way tables.
- Interpret bar charts, pie charts, grouped frequency diagrams, line graphs, and frequency polygons.
- Interpret cumulative frequency tables and graphs, box plots, and histograms (including unequal class width).
- Calculate and interpret mean, median, mode, modal class, range, and inter-quartile range, including the estimated mean of grouped data.
- Calculate average rates when combining samples or events, including solving problems involving average rate calculations (e.g. average survival rates in different wards of different sizes, average speed of a car over a journey where it has travelled at different speeds).
- Interpret scatter diagrams and recognise correlation; using lines of best fit. (The calculation of regression lines is not required.)
- Compare sets of data by using statistical measures or by interpreting graphical representations of their distributions.

# Probability

- Understand and use the vocabulary of probability and the probability scale.
- Understand and use estimates or measures of probability, including relative frequency and theoretical models.
- List all the outcomes for single and combined events.
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1.
- Construct and use Venn diagrams to solve union and intersection categorisation problems and determine probabilities when required. Familiarity with the meaning and use of the terms 'union', 'intersection', and 'complement' is required. The mathematical notation for these ($A \cup B$, $A \cap B$, and $A'$ or $A^c$) will not be required.
- Know when to add or multiply two probabilities.
- Understand the use of tree diagrams to represent outcomes of combined events:
  - when the probabilities are independent of the previous outcome;
  - when the probabilities are dependent on the previous outcome.
- Compare experimental and theoretical probabilities.
- Understand that if an experiment is repeated, the outcome may be different.

# SECTION 2

*This section sets out the scope of Paper 2. Paper 2 tests the candidate's ability to think mathematically: the paper will focus on testing the candidate's ability to understand, and construct, mathematical arguments in a variety of contexts. It will draw on the mathematical knowledge outlined in* **SECTION 1** *above.*

## The Logic of Arguments

**Arg1** Understand and be able to use mathematical logic in simple situations:

- The terms **true** and **false**;
- The terms **and**, **or** (meaning **inclusive or**), **not**;
- Statements of the form:

  **if A then B**
  **A if B**
  **A only if B**
  **A if and only if B**

- The **converse** of a statement;
- The **contrapositive** of a statement;
- The relationship between the truth of a statement and its converse and its contrapositive.

**Note**: candidates will **not** be expected to recognise or use symbolic notation for any of these terms, nor will they be expected to complete formal truth tables.

**Arg2** Understand and use the terms **necessary** and **sufficient**.

**Arg3** Understand and use the terms **for all**, **for some** (meaning **for at least one**), and **there exists**.

**Arg4** Be able to negate statements that use any of the above terms.

## Mathematical Proof

**Prf1** Follow a proof of the following types, and in simple cases know how to construct such a proof:

- Direct deductive proof ('Since A, therefore B, therefore C, …, therefore Z, which is what we wanted to prove.');
- Proof by cases (for example, by considering even and odd cases separately);
- Proof by contradiction;
- Disproof by counterexample.

**Prf2** Deduce implications from given statements.

**Prf3** Make conjectures based on small cases, and then justify these conjectures.

**Prf4** Rearrange a sequence of statements into the correct order to give a proof for a statement.

**Prf5** Problems requiring a sophisticated chain of reasoning to solve.

**Identifying Errors in Proofs**

**Err1**    Identifying errors in purported proofs.

**Err2**    Be aware of common mathematical errors in purported proofs; for example, claiming 'if $ab = ac$, then $b = c$' or assuming 'if $\sin A = \sin B$, then $A = B$' neither of which are valid deductions.